Rocket Motor Service Life Calculations Based on the First-Passage Method

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The paper discusses the uses of the first-passage method in service life predictions for solid propellant rocket motors. These motors, when stored at a site for a long period of time, are subjected to environmental temperature variations, aging, and cumulative damage. Mechanical properties are considered as statistically variable quantities. Temperature variations treated alternatively as Poisson and Markov processes are compared under cold and warm climates. As expected, service lives of motors were found to be shorter at the cold site. The Markov model gives somewhat more conservative results than the Poisson model.

Nomenclature

A,B	= aging parameters
a	= Raleigh distributed amplitude
a_T	= visco-elastic shift function
$b(t)$	= probabilistic strength, psi (N/m^2)
C	= damage parameter
D_n	= cumulative damage
$E_i,E_r,E(\omega)$, $E'(\omega),E''(\omega)$	= moduli, for Maxwell elements, relaxation, complex, storage, loss, psi (N/m^2)
$E[\cdot]$	= expected value
$F(\tau)$	= first passage probability
G_{in}	= input power spectrum, °F^2/(rad/h) (°C^2/rad/h)
G_{jo}	= output power spectrum, °F^2/(rad/h) (°C^2/rad/h)
i	= index for time steps
j	= index for jth layer
k	= thermal conductivity, Btu/ft h, °C(W/m°k)
m	= number of events in interval (θ,τ)
$n(t)$	= crossing rate
P,P_f	= probability, probability of failure
$Q(t)$	= random variation of strength, psi (N/m^2)
R,R'	= mean virgin strength, reduced strength, psi (N/m^2)
$R(r,\omega)$	= temperature frequency response function, °F/°F (°C/°C)
r	= radial coordinate
$S_r,S_\theta,S(t),S_d,S_y$	= stress; radial, tangential, total, daily amplitude, annual amplitude, psi (N/m^2)
$T(r,t),T_f$	= temperature, stress-free temperature, °F(°C)
t,t_f	= time, time to failure due to damage
$w(T)$	= noise-induced thermal stress psi (N/m^2)
$x(t)$	= stochastic process
$y(t)$	= time-varying barrier
z	= ratio
α	= thermal diffusity, in.^{2}/h (m^2/h)
$\alpha_m(\tau)$	= accumulated Markov crossing rate
$\alpha_p(\tau)$	= accumulated Poisson crossing rate
β	= damage parameter
ΔT	= time step = 1 h
δ_D,δ_R	= coefficient of variation; strength, damage
η	= aging factor
$\Phi(\cdot)$	= normal probability function
$\phi(\cdot)$	= normal density function
ϕ_d,ϕ_y	= diurnal and annual phase angle
ρ	= aging parameter
μ_T,μ_y	= mean temperatures
$\sigma_b,\sigma_D,\sigma_k,\sigma_w,\sigma_x,\sigma_{\dot{x}}$	= standard deviations of the respective processes
τ	= time interval
τ_r	= first passage time
τ_i	= relaxation times for Maxwell elements
θ	= random phase angle
ω_d,ω_y	= diurnal and annual frequency, rad/h

Presented as Paper 86-1416 at the AIAA/ASME/SAE/ASEE 22nd Joint Propulsion Conference, Huntsville, AL, June 16-18, 1986; received June 28, 1988; revision received Jan. 4, 1989.

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Introduction

SOLID propellant rocket motors are idealized as long, hollow, layered, viscoelastic cylinders. They are affected by environmental temperature changes that produce both variable mechanical properties and cyclic thermal stresses. These thermal stresses may reach levels at which the motor no longer performs its objective with a high level of reliability. Therefore, the problem of service life prediction for solid propellant rocket motors is of interest. Because environmental temperature is considered to be a random process, a stochastic approach is applicable to this problem.

In a recent paper,[1] the authors presented the concept of first passage to determine the reliability of thermally loaded cylindrical structures. Contrary to the familiar assumptions of constant deterministic barriers, statistical and time-varying barriers and a Poisson process were assumed. Here, a continuation is presented. A comparison between two geographic locations with extreme weather conditions is shown. Results based on both the Poisson and Markov models are discussed for the random temperature process, and aging effects are shown.

Table 1 Geometrical and physical properties

	Layer 1	Layer 2	Layer 3	Layer 4
Radius, in. (m)	1.875 (0.048)	4.3 (0.109)	4.4 (0.112)	6.6 (0.165)
Thermal conductivity, Btu/h-ft-°F (W/m-°C)	0.015 (0.025)	0.76 (1.322)	14.6 (25.25)	0.016 (0.027)
Diffusivity, ft²/h (m²/s)	0.763 (0.197 E − 04)	0.030 (0.785 E − 06)	0.34 (0.877 E − 05)	8.575 E − 03 (2.214 E − 07)
Coefficient of thermal expansion, in./in./°F (m/m/°C)		5.78 E − 05 (1.04 E − 04)	6.5 E − 07 (1.17 E − 05)	
Modulus of elasticity psi (N/m²)		281.84 (1.943 E+06)	30 E − 06 (2.068 E+11)	
Poisson's ratio		0.49	0.25	
Stress-free temperature °F (°C)	165 (74)			
Damage parameters stress in psi (N/m²)	$\beta = 8.75$ $C = 1.421$ E + 16 (5.491 E + 49)			
Modulus aging parameters, temperature in °R (K)	$A = 4.1$ E + 05 $B = 8.73$ E + 03 (4.85 E + 03)			
Strength aging parameters, temperature in °R (K)	$A = 1.15$ E + 10 $B = 1.53$ E + 04 (8.53 E + 03)			

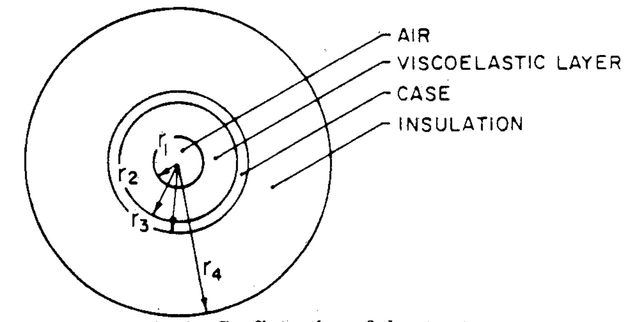

Fig. 1 Configuration of the structure.

Heat-Transfer Problem

The rocket motor is idealized as an axisymmetric, layered cylinder as shown in Fig. 1 and is treated as a one-dimensional problem. Physical and geometric parameters are given in Table 1. The random thermal load is the air temperature that was obtained from temperature-time records at the two sites of interest. This time series can be separated into four components: mean, annual cycle, diurnal cycle, and noise. The noise is modeled as a stationary random process whose power spectrum $G_{\text{in}}(\omega)$ can be calculated by means of fast Fourier transforms. The method of solution requires determination of the frequency response function, which can be obtained by solving the heat-conduction equation in cylindrical coordinates,

$$\frac{\partial^2 T_j}{\partial r^2} + \frac{1}{r}\frac{\partial T_j}{\partial r} = \frac{1}{\alpha_j}\frac{\partial T_j}{\partial t} \tag{1}$$

The temperature of the jth layer, $T_j(r,t)$, is a function of both the radial coordinate r and time t, with α_j the thermal diffusivity of the jth layer. For a four-layer cylinder, the boundary conditions (BC) are as follows:

BC 1: $T_1(0,t)$; temperature at the center is finite.

BC 2-4: $T_j(r,t) = T_{j+1}(r,t)$; temperatures are the same on both sides of an interface.

BC 5-7: $k_j[\partial T_j(r,t)]/(\partial r) = k_{j+1}[\partial T_{j+1}(r,t)]/(\partial r)$; the heat flux across an interface is continuous.

BC 8: $T_4(r_4,t) = e^{i\omega t}$; the temperature on the surface of the fourth layer varies sinusoidally with a unit amplitude and a frequency ω.

The temperature at an interior point in the propellant can be written as[2]

$$T_j(r,t) = |R_j(r,\omega)|\cos(\phi_j + \omega t) \tag{2}$$

where $R_j(r,\omega)$ is the frequency response function given in terms of complex Kelvin functions, and ϕ_j is the phase angle.

The power spectrum $G_{jo}(r,\omega)$ of temperatures at an internal point produced by the input noise is related to the input power spectrum as

$$G_{jo}(r,\omega) = |R_j(r,\omega)|^2 G_{\text{in}}(\omega) \tag{3}$$

The mean square value of the temperature at an internal point is given as the area under the output power spectral density function.

$$E[T_j^2(r)] = \int_{-\infty}^{\infty} |R_j(r,\omega)|^2 G_{\text{in}}(\omega)\,\mathrm{d}\omega \tag{4}$$

Thermal Stresses and Viscoelastic Analysis

As a consequence of Eq. (4), the mean square value of the thermal stresses is obtained as

$$E[S_k^2(r,\omega)] = \int_{-\infty}^{\infty} |S_k(r,\omega)|^2 G_{\text{in}}(\omega)\,\mathrm{d}\omega \tag{5}$$

where $S_k(r,\omega)$ is the frequency response function of either the radial or tangential stress component. These are given as[3]

$$S_r(r,\omega) = \frac{r_2^2 p'}{r_2^2 - r_1^2}\left(1 - \frac{r_1^2}{r^2}\right) + \frac{\bar{\alpha}_2 E_2}{(1-\upsilon_2)(r_2^2 - r_1^2)} \times \left(1 - \frac{r_1^2}{r^2}\right)\int_{r_1}^{r_2} R_2(r,\omega) r\,\mathrm{d}r - \frac{\bar{\alpha}_2 E_2}{(1-\upsilon_2)r^2} \times \int_{r_1}^{r} R_2(r,\omega) r\,\mathrm{d}r \tag{6}$$

$$S_\theta(r,\omega) = \frac{-r_2^2 p'}{r_2^2 - r_1^2}\left(1 - \frac{r_1^2}{r^2}\right) + \frac{\bar{\alpha}_2 E_2}{(1-\upsilon_2)(r_2^2 - r_1^2)} \times \left(1 - \frac{r_1^2}{r^2}\right)\int_{r_1}^{r_2} R_2(r,\omega) r\,\mathrm{d}r + \frac{\bar{\alpha}_2 E_2}{(1-\upsilon_2)r^2} \times \int_{r_1}^{r} R_2(r,\omega) r\,\mathrm{d}r - \frac{\bar{\alpha}_2 E_2 R_2(r,\omega)}{(1-\upsilon_2)} \tag{7}$$

where

$$p' = \frac{E_2[2\bar{\alpha}_2(1+\upsilon_2)\int_{r_1}^{r_2} R_2(r,\omega)r\,\mathrm{d}r - \bar{\alpha}_3(1+\upsilon_3)(r_2^2 - r_1^2)R_2(r_2,\omega)]}{(1+\upsilon_2)[(1-2\upsilon_2)r_2^2 + r_1^2] + (E_2/E_3)(1-\upsilon_3^2)r_2[(r_2^2-r_1^2)/(r_3-r_2)]} \tag{8}$$

Service life calculations are based on the critical induced thermal stress $S(t)$, which is the tangential component at the bore. It is modeled as the sum of four terms.

$$S(t) = \mu(t) + S_y\cos(\omega_y t + \phi_y) + S_d\cos(\omega_d t + \phi_d) + w(t) \tag{9}$$

where $\mu(t)$ is the mean stress induced by the difference between the stress-free temperature and the mean temperature; S_y and S_d are annual and diurnal amplitudes of thermal stress, which are related to the respective temperature amplitudes; ω_y and ω_d are yearly and daily frequencies in radians per hour; ϕ_y and ϕ_d are the yearly and daily phase angles; and $w(t)$ is the random component, the result of thermal noise.

The mean stress value is obtained by substituting the difference between the stress-free temperature T_f and the yearly mean temperature μ_y

$$\mu_T = -(T_f - \mu_y) \tag{10}$$

in place of $R_2(r,\omega)$ in Eqs. (6-8). The yearly and daily amplitudes of thermal stresses are calculated by multiplying the absolute values of the frequency response function at the yearly and daily frequencies with the amplitudes of corresponding cyclic temperatures. The viscoelastic effects are introduced by replacing the elastic modulus with the complex modulus[4,5]

$$E(\omega) = E'(\omega) + iE''(\omega) \tag{11}$$

with $E'(\omega)$, the storage modulus and $E''(\omega)$ the loss modulus. These are obtained through Fourier transforms of the relaxation modulus $E_r(t)$:

$$E_r(t) = \sum_{i=1}^{\ell} E_i e^{-t/\tau_i a_T} \tag{12}$$

E_i and τ_i are the moduli and relaxation times of parallel Maxwell elements, and a_T is the viscoelastic shift function

$$\log a_T = 1.42\left[\frac{-8.86(T+34)}{181.9+(T+34)} + 3.32\right] \tag{13}$$

The master curve for the mean viscoelastic strength $\overline{R}_0(t)$ may be represented by an equation of the type[4]

$$\log\overline{R}_0(t) = 2.1430 - .0857\log\frac{t}{a_T} \tag{14}$$

where $\overline{R}_0$ is the mean strength in psi, and t is the time in minutes.

The linear cumulative damage rule proposed by Bills can be written as[6]

$$D_n = \sum_{i=1}^{n}\frac{\Delta t}{t_{f_i}} \tag{15}$$

where t_{f_i} is the time required to produce failure, and $\Delta t = 1$ h. A relationship between constant stress and reduced time to failure is given as

$$\frac{t_{f_i}}{a_{T_i}} = CS_i^{-\beta} \tag{16}$$

where C and β are material parameters listed in Table 1. Hence, cumulative damage after n time steps becomes

$$D_n = \frac{1}{C}\sum_{i=1}^{n}\frac{\Delta t S_i^{\beta}}{a_{T_i}} \tag{17}$$

Although damage always increases with time, the time-temperature shift function a_{T_i} in Eq. (17) produces a variable damage rate due to the random temperature input. Because of the statistical variability of the damage parameter C, a coefficient of variation δ_D is assumed.

Additionally, a time- and temperature-dependent hardening or softening of the voscoelastic material, called aging, defined as the change in the physical parameters of the material in an unloaded condition, also takes place. The two parameters that are affected by aging are the modulus and strength. The aging factor for a parameter, modulus or strength, is written as[7]

$$\eta(T,t) = 1_{-\rho}\log t \tag{18}$$

where

$$\rho = Ae^{-B/T} \tag{19}$$

in which A and B are material parameters given in Table 1, and T is the absolute temperature.

The mean residual strength of the material can be written as[6]

$$\overline{R}' = \bar{\eta}_R\overline{R}_0(1-\overline{D}) = \overline{R}(1-\overline{D}) \tag{20}$$

where $\overline{R}_0$ is the mean virgin strength adjusted for viscoelastic effects, $\bar{\eta}_R$ is the average strength aging factor, and $\overline{D}$ is the mean value of damage. The standard deviation of the residual strength is evaluated from those of the strength σ_R and the cumulative damage σ_D.

$$\sigma_{R'}^2 = \overline{R}^2\sigma_D^2 + \sigma_R^2(1-2\overline{D}+\overline{D}^2+\sigma_D^2) \tag{21}$$

First-Passage Analysis

Of major interest in studying the service life of structures is the problem of determining the time when a prescribed response such as stress or displacement first passes out of a limited domain of safe operation (level or barrier) with a given probability. When this probability exceeds a certain specified value, the structure is said to have a probability of first-excursion (or passage) failure as opposed to other types of failure. The study of this phenomenon is known in the literature as first-passage problem.[8-11] Information on the time to first passage is of great practical importance. It describes the reliability or quality of performance of a structure subjected to certain kinds of random loads. This problem has been discussed in Refs. 1 and 12 for different cases.

The probabilty of failure for the system can be written as

$$P_f = P[S(t) \geq b(t)] \tag{22}$$

where $S(t)$ is the induced stress given in Eq. (9), and $b(t)$ is the probabilistic strength that consists of two parts

$$b(t) = \overline{R}'(t) + Q(t) \tag{23}$$

in which $\overline{R}'(t)$ is the reduced mean strength given in Eq. (20), and $Q(t)$ is the random variation of the damaged strength

$$Q(t) = a\cos(\omega t + \theta) \tag{24}$$

$Q(t)$ is a normal stochastic narrow-band process where ω is assumed to be the yearly frequency, θ is uniformly distributed over the range $(0,2\pi)$, and a is Rayleigh distributed. Separating the deterministic components from the stochastic ones, Eq. (22) becomes

$$P_f = P[x(t) \geq y(t)] \tag{25}$$

where $x(t)$ is the stochastic process

$$x(t) = w(t) - Q(t) \tag{26}$$

and $y(t)$ is the time-varying barrier

$$y(t) = \overline{R}'(t) - \mu(t) - S\cos(\omega_y t + \phi_y) - S_d \cos(\omega_d t + \phi_d) \tag{27}$$

This reduces the problem of crossing a random barrier by a stochastic process to that of crossing a curve by a different stochastic process.

Assuming the noise, the strength, and their velocity processes to be independent, the variances of the modified stochastic process and its velocity process can be written as

$$\sigma_x^2 = \sigma_w^2 + \sigma_b^2 \tag{28}$$

$$\sigma_{\dot{x}}^2 = \sigma_{\dot{w}}^2 + \omega_y^2 \sigma_b^2 \tag{29}$$

The variances of the noise σ_w^2 and its velocity process $\sigma_{\dot{w}}^2$ are given as[13]

$$\sigma_w^2 = \int_{-\infty}^{\infty} | S(r_1,\omega) |^2 G_{\text{in}}(\omega)\, d\omega \tag{30}$$

$$\sigma_{\dot{w}}^2 = \int_{-\infty}^{\infty} \omega^2 | S(r_1,\omega) |^2 G_{\text{in}}(\omega)\, d\omega \tag{31}$$

The crossing rate according to Rice becomes

$$n(t) = \frac{2\sigma_{\dot{x}}}{\sigma_x}\phi(0)\phi\left(\frac{y}{\sigma_x}\right) \tag{32}$$

It also can be written more rigorously as[14]

$$n(t) = \frac{1}{\sigma_x}\phi\left(\frac{y}{\sigma_x}\right)\left\{2\sigma_{\dot{x}}\left(\frac{\dot{y}}{\sigma_{\dot{x}}}\right) + \dot{y}\left[2\Phi\left(\frac{\dot{y}}{\sigma_{\dot{x}}}\right) - 1\right]\right\} \tag{33}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and cumulative functions of the normal distribution, σ_x and $\sigma_{\dot{x}}$ are the standard deviations of $x(t)$ and its velocity process. Comparing the preceding two expressions for the case of a linear barrier of the form

$$y(t) = a + bt \tag{34}$$

one would write

$$z = \exp\left(\frac{-b^2}{2\sigma_{\dot{x}}^2}\right) + \frac{\sqrt{2\pi}}{2\sigma_{\dot{x}}}b\left[2\Phi\left(\frac{b}{\sigma_{\dot{x}}}\right) - 1\right] \tag{35}$$

for the ratio of the two expressions (32) and (33). The variation of this ratio as a function of $b/\sigma_{\dot{x}}$ indicates that the ratio of the two equations is significant whenever the rate of change of the barrier is high. This ratio for the problem at hand is nearly unity. Had there been a higher rate of change of the barrier due to higher frequency, or faster degradation of the strength by aging or damage, the effect of Eq. (33) would have been more pronounced.

The crossings of the barrier are assumed to occur according to a Poisson or a Markov process. In the case of the Poisson process, the probability function is written as

$$P(m,t) = \frac{(n\tau)^m}{m!}\exp(-n\tau) \tag{36}$$

where m is the number of events in the interval $(0,\tau)$, and n is again the crossing rate. The probability of no events, or zero number of crossings, becomes

$$P(0,t) = \exp(-n\tau) \tag{37}$$

In general, this equation can be given as

$$P(0,t) = \exp[-\alpha_p(\tau)] \tag{38}$$

where

$$\alpha_p(\tau) = \int_0^\tau n(t)\, dt \tag{39}$$

Equation (38) is equivalent to the statement that the first passage time τ_l is greater than τ. The first passage time is defined here as the time at which $y(t)$ is crossed for the first time. The probability of first passage, which is the probability of at least one up-crossing in the interval $(0,\tau)$, is given by

$$F(\tau) = 1 - \exp[-\alpha_p(\tau)] \tag{40}$$

Following Vanmarcke,[12] crossings are also considered as a two-state Markov process. That is, the process is visualized as passing randomly back and forth from state 0, below the barrier, to state 1, above the barrier. The time $(\tau_0 + \tau_1)$ is the time between successive up-crossings. It is related to Eqs. (32) and (33) as

$$E[\tau_0 + \tau_1] = \frac{1}{n(t)} \tag{41}$$

The time intervals τ_0 and τ_1 are the intervals that the process spends below and above the barrier, respectively. They are assumed to be independent and exponentially distributed. The reciprocal of their rates $E[\tau_0]$ and $E[\tau_1]$ can be found by applying the continuous Markov theory. They can be written as

$$E[\tau_0] = \frac{1}{n(t)}\Phi\left(\frac{y}{\sigma_x}\right) \tag{42}$$

$$E[\tau_1] = \frac{1}{n(t)}\left[1 - \Phi\left(\frac{y}{\sigma_x}\right)\right] \tag{43}$$

In light of Eqs. (41-43), a modified expression for the first passage probability can be given as

$$F(\tau) = 1 - L(0)\exp[-\alpha_m(\tau)] \tag{44}$$

where $L(0)$ is the probability that the process starts in the safe region

$$L(0) = \frac{E[\tau_0]}{E[\tau_0 + t_1]} = \Phi\left(\frac{y}{\sigma_x}\right) \tag{45}$$

and $\alpha_m(\tau)$ is given as

$$\alpha_m(\tau) = \int_0^\tau \frac{1}{E[\tau_0]}\, dt \tag{46}$$

An alternate aproach to the probabilistic variation of strength can be obtained by considering the strength as a random variable $b(t)$ with mean $\overline{R}'(t)$ given in Eq. (20) and standard deviation $\sigma_b(t)$ given in Eq. (21). The stochastic process $x(t)$ of Eq. (26) is in this case just that of the noise of thermal stress $w(t)$.

The total first-passage probabilty for a probabilistic barrier becomes

$$F_w(\tau) = \int_{-\infty}^{\infty} F_{W|B}(\tau) f_B(b)\, db \tag{47}$$

where $F_{W|B}(\tau)$ is a conditional probability

$$F_{W|B}(\tau) = 1 - \exp[-\alpha(\tau)] \tag{48}$$

in which $\alpha(\tau)$ is either $\alpha_m(\tau)$ or $\alpha_p(\tau)$, and $f_B(b)$ is the density function of the strength. The strength is assumed to follow one of three distribution functions: normal, log-normal, or Weibull distribution. The details of this treatment were presented in Refs. 1 and 13.

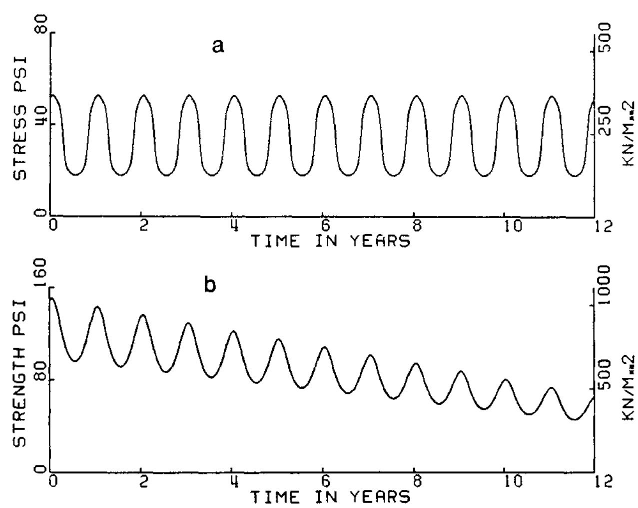

Fig. 2 a) Mean stress and b) mean strength at a cold site; cumulative damage is included.

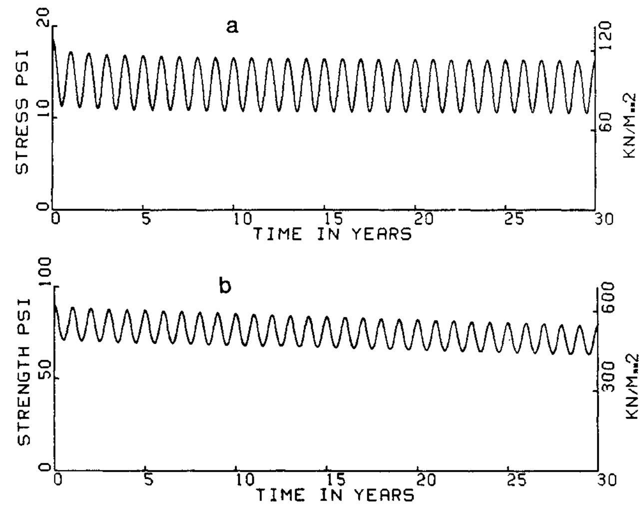

Fig. 3 a) Mean stress and b) mean strength at a warm site; cumulative damage and aging are included.

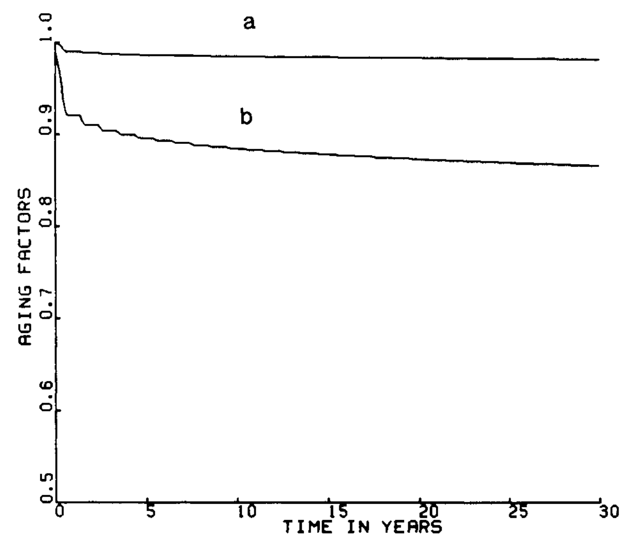

Fig. 4 a) Strength and b) modulus aging factors at a warm site.

A special case of Eq. (47) is the case of a constant barrier and normal strength; rewriting Eq. (47)

$$F_w(\tau) = 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left\{ -\frac{1}{2\pi} \frac{\sigma_{\dot{w}}}{\sigma_w} \times e^{-\frac{1}{2}\left[\frac{\sqrt{2}u\sigma_b + \bar{R}'(t)}{\sigma_w}\right]^2} \tau \right\} \times e^{-u^2} \mathrm{d}u \tag{49}$$

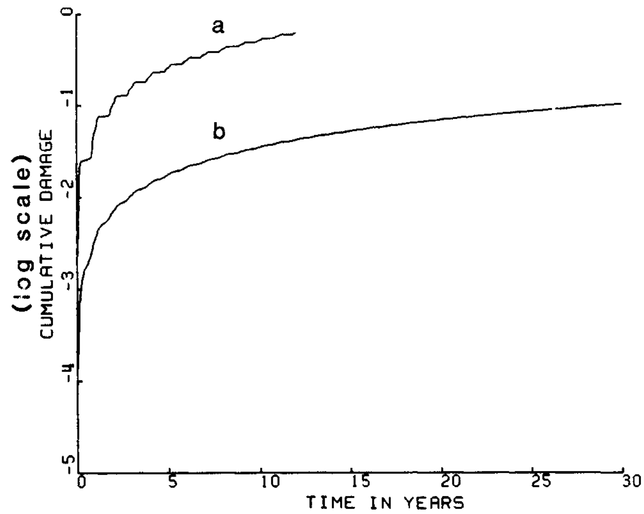

Fig. 5 Cumulative damage at a) the cold and b) the warm location.

where

$$u = \frac{1}{\sqrt{2}} \left[\frac{b - \bar{R}'(t)}{b} \right] \tag{50}$$

For a small crossing rate, Eq. (49) becomes

$$F_w(\tau) = 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left\{ 1 - \frac{1}{2\pi} \frac{\sigma_{\dot{w}}}{\sigma_w} \times \tau e^{-\frac{1}{2}\left[\frac{\sqrt{2}u\sigma_b + \bar{R}'(t)}{\sigma_w}\right]^2} \right\} \times e^{-u^2} \mathrm{d}u \tag{51}$$

The integration in Eq. (51) yields

$$F_w(\tau) = \frac{1}{2\pi} \frac{\sigma_{\dot{w}}}{(\sigma_b^2 + \sigma_w^2)^{1/2}} \exp\left[\frac{-b^2}{2(\sigma_b^2 + \sigma_w^2)} \right] \tag{52}$$

An interesting resemblance between Eqs. (52) and (32) is clear.

Discussion of Results

Numerical calculations were carried out for different periods of time at two sites that represent two extreme climates. Figures 2a and 2b and 3a and 3b show the variations of thermal stresses and strengths for 12 years at a cold location, and for 30 years at a warm site, respectively. The oscillations in the thermal stresses and strengths are due to the viscoelastic nature of the propellant materials. When the temperature decreases in the winter season, the modulus becomes higher and thus higher thermal stresses are induced. Strength also increases with reduced temperatures. The reverse is true when the temperature increases. The reductions in the stress and strength due to aging and cumulative damage are included. Figure 4 shows the difference between aging factors for the modulus and strength. A comparison between damage effects at the two sites is shown in Fig. 5. As expected, damage effects are more severe at the cold site than they are at the warm one.

The corresponding crossing rates and first-passage probabilities are shown in Figs. 6a and 6b and 7a and 7b. The coefficient of variation of the strength δ_R is equal to 10%, and the coefficient of variation of damage δ_D is taken as 20%. Comparing the last two sets of figures, it is seen that first-passage probabilities at the cold site reach a value of 1 after a period of 8 years, whereas the motor at the other location is quite safe after 30 years. The reason is that the thermal stresses are comparatively high at the cold site, and cumulative damage reduces the strength to about 50% of its initial value in 10 years.

A comparison between the Poisson approach and Markov approach is shown in Figs. 8 and 9. The higher curve in Fig. 8 corresponds to the Markov crossing rate. Because the

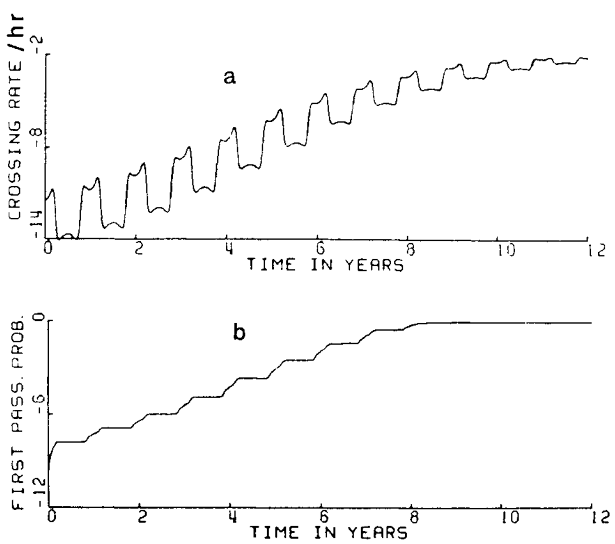

Fig. 6 a) Crossing rate and b) first-passage probability at the cold site with $\delta_R = 0.1$, $\delta_D = 0.2$.

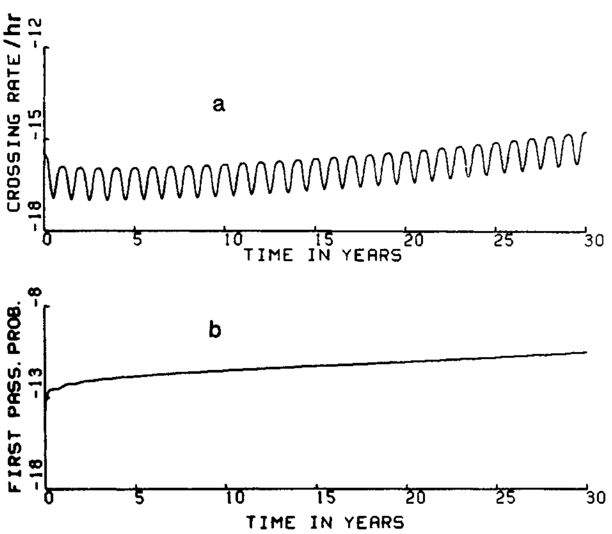

Fig. 7 a) Crossing rate and b) first-passage probability at the warm site with $\delta_R = 0.1$, $\delta_D = 0.2$; aging is included.

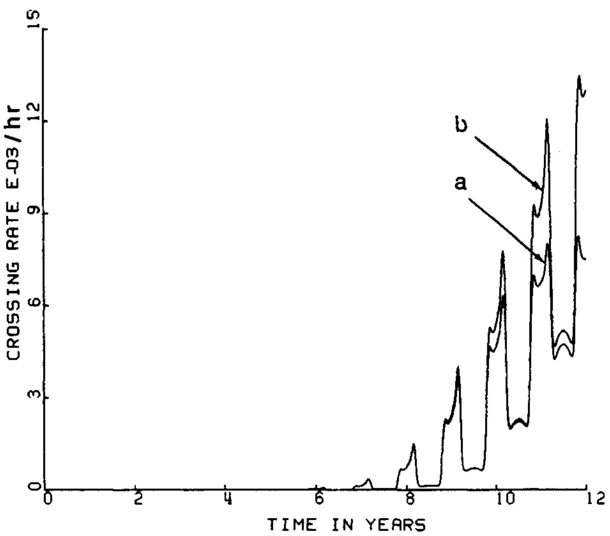

Fig. 8 Comparison between a) Poisson [see Eq. (33)] and b) Markov [see Eq. (42)] assumptions.

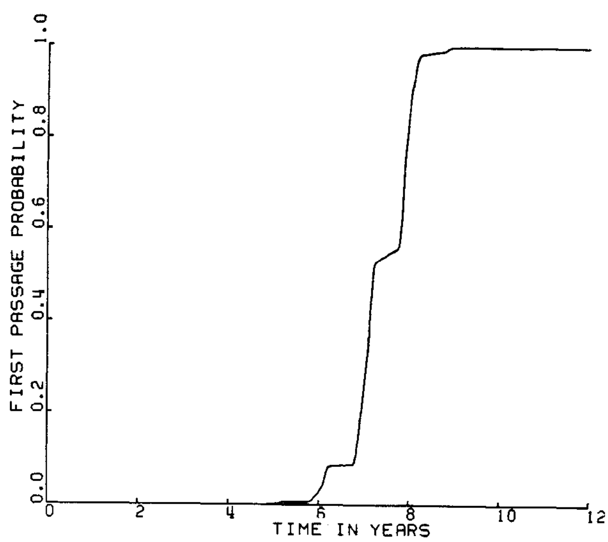

Fig. 9 First-passage probability at the cold site. The Poisson and Markov processes are indistinguishable; the two curves overlap.

Markov effect becomes significant only at a late stage in the life of the structure, the Poisson and Markov first-passage probabilities overlap as shown in Fig. 10. Based on the crossing rates, it may be stated that the Markov model is somewhat more conservative.

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